Quantum Chaos in Helium

Transition to Chaos Observed in a Simple Quantum System

Two's a party, three's a crowd—especially in a tiny space. Two objects that exert electrostatic or gravitational forces on each other have relatively simple dynamics: the forces scale as the square of the distance between the objects. A three-body system, however, cannot be solved analytically (it is nonintegrable), which indicates that the dynamics involve a mixture of regularity and chaos. Add to that the constraints of quantum mechanics, and things get truly challenging. Now, an investigation into the transition from quantum dynamics to chaos in the spectrum of helium has shed a little bit of light on one of physics's blackest boxes—quantum chaos.

Since the work of Poincaré a century ago, the problem of three bodies interacting under their mutual gravitational forces (such as the earth, moon, and sun) has been known to exhibit a mixture of classical and chaotic dynamics. A system of three charged particles should have similar dynamics (within a sign), even in very small systems. But as yet, scientists don't know how to reconcile chaos with a quantum mechanical view of the universe. Classical dynamics allows for the chaotic motion of three bodies, because the mechanics can be described with nonlinear equations of motion; quantum mechanics, however, does not have this way to account for chaos, because the Schrödinger equation is linear. Furthermore, the quantum states of the helium atom, the prototypical three-body charged-particle system, occur in seemingly regular progressions, labeled by sets of quantum numbers. How, then, can classical mechanics and quantum mechanics be reconciled? What are the manifestations of the underlying classical chaos in the quantum spectrum of helium?

To answer these questions, an international group working at Beamline 9.0.1 (now Beamline 10.0.1) has used the bright beams of the ALS along with theoretical modeling to search for quantum chaos in the photoabsorption spectrum of helium—and they've found it. The high brightness on this undulator beamline allowed the resolution (about 2 meV) necessary to distinguish tightly spaced states near helium's double-ionization threshold. Electrons in these high-energy, doubly excited states are known to show more classical behavior than those lying closer to the nucleus. But the states are so close together that a third-generation light source is needed to resolve them. The resulting spectrum was compared to a new theoretical model based on a random matrix approach to chaotic systems. Agreement between the model and the data was excellent, allowing the experimenters to extend the statistical analysis even to states above those seen in the experimental spectrum.

The electronic states of doubly excited helium can be labeled as N,K_n, where N is the principal quantum number of the inner electron, n is that of the outer electron, and K is the angular correlation between the two. States with the same N converge to an ionization threshold, I_N. The researchers found that, as the electron energies approach the ionization threshold for electrons with higher N, the statistical properties of the spacing between neighboring energy levels clearly display a transition toward quantum chaos. Where $I_N > 4$, the N–1 series begins to be perturbed by higher series. Where $I_N > 8$, the effect is strong enough that traditional quantum numbers can no longer describe the dynamics. Statistical analyses also showed that, as I_N increases, plots of the spacings between nearest-neighbor states move from being best described by a Poisson distribution (associated with regular systems) to more closely approximating a Wigner distribution (associated with chaotic systems). This observation of the onset of chaotic dynamics in a simple three-body system shows, for the first time, how the underlying classical chaos manifests in a simple and well-studied quantum system. ■

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R. Püttner, B. Grémaud, D. Delande, M. Domke, M. Martins, A.S. Schlachter, and G. Kaindl, "Statistical Properties of Inter-Series Mixing in Helium: From Integrability to Chaos," *Phys. Rev. Lett.* **86**, 3747 (2001).

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QUANTUM CHAOS IN HELIUM

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- **Classic three-body problem**
  - *Two-body system: forces scale as $1/r^2$*
  - *Three-body system: mixture of regular and chaotic dynamics*
- **How can quantum mechanics work with classical chaos?**
  - *Classical mechanics is compatible with chaos theory via nonlinear equations of motion*
  - *Schroedinger's equation is linear!*
- **Observation of transition from quantum to chaotic behavior in helium photoabsorption spectrum**
  - *Prototypical three-body quantum system (one nucleus, two electrons)*
  - *Third-generation light source needed to resolve states near double-ionization threshold*
  - *Statistical analysis of nearest-neighbor spacings between electron states*
  - *Transition from Poisson distribution (non-chaotic) to Wigner distribution (chaotic)*

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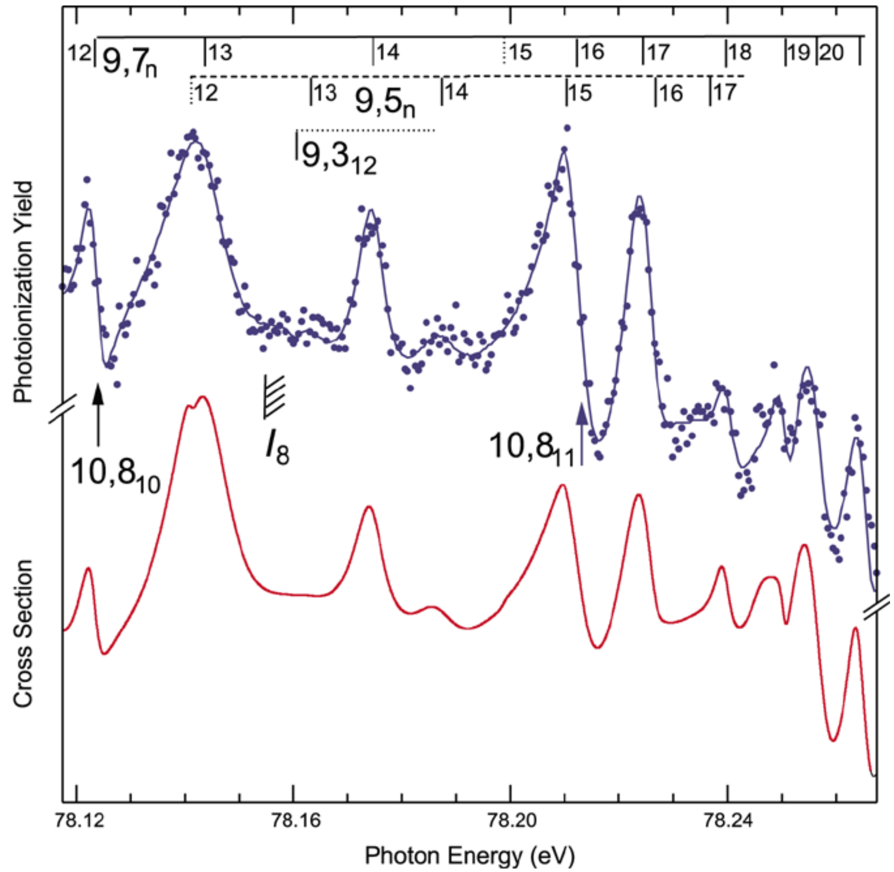

Blue, *photoabsorption spectrum of helium gas after double-electron excitation. The solid line through the data points indicates the best fit.* Red, *spectrum calculated from theoretical model.*

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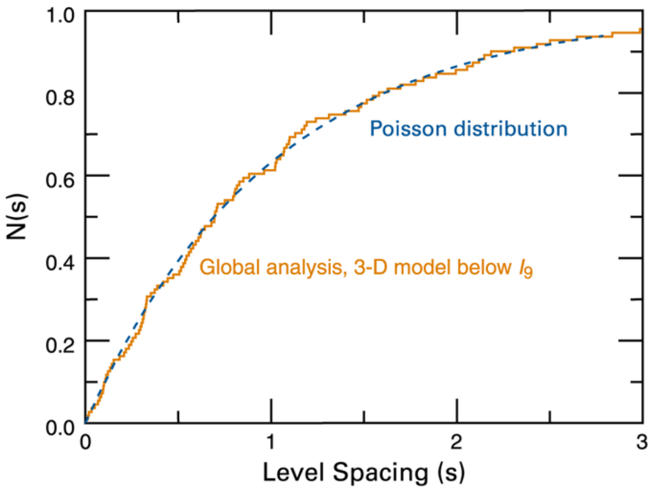
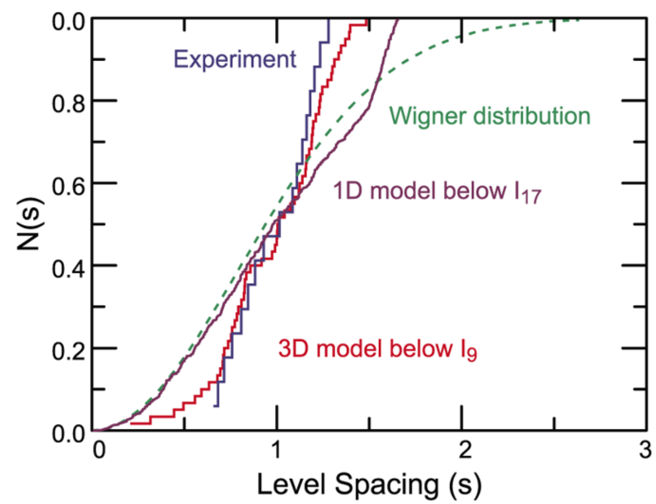


Cumulative distribution of nearest-neighbor spacings for the $^1P^o$ states of helium below I_9. The data agree very well with a cumulative Poisson distribution (dashed blue line), which is indicative of a non-chaotic system.

Cumulative distributions of nearest-neighbor spacings for the $^1P^o$ states of helium, analyzed individually for each series associated with a given value of N – K: blue, *distribution derived from experiment;* red, *best fit from three-dimensional model calculations for states below I_9;* purple, *best fit from one-dimensional model calculations for states below I_{17};* dashed green line, *Wigner distribution. The marked tendency toward a Wigner distribution as higher lying states are included indicates the onset of chaos.*



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